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Problem set 1

Exercise 1
Suppose the process for income is

$$y_t = \overline{y} + \varepsilon_t + \beta\varepsilon_{t-1} \tag{1}$$

Using the permanent income, 3-period model studied in class, find the expression for $c_2 - c_1$ as function of $\varepsilon_{2.}$ and ε_1.

Exercise 2
Use the permanent income, 3-period model studied in class, and the stochastic process $y_{t+1} = \lambda y_t + \varepsilon_{t+1}$. Show the consumption function linking c_1 to its determinants y_1 and A_1. What happens if $\lambda = 1$ or $\lambda = 0$?

Exercise 3
Consider the two-period model

$$\max_{c_1,c_2} U = u(c_1) + \frac{1}{1+\rho}u(c_2) \tag{2}$$

s.t.

$$c_1 + \frac{c_2}{1+r} = y_1 + \frac{y_2}{1+r} \tag{3}$$

The utility function is

$$u(c) = \frac{c^{1-\sigma} - 1}{1-\sigma} \quad \text{for } \sigma \geq 0, \quad \sigma \neq 1 \tag{4}$$

$$u(c) = \log c \quad \text{for } \sigma = 1 \tag{5}$$

a) Write the Lagrangean of the problem and show that λ (the Lagrange multplier on the intertemporal budget constraint) is equal to the marginal utility of consumption in period 1.

b) Show that the Euler equation is

$$\left(\frac{c_1}{c_2}\right)^\sigma = \frac{1+\rho}{1+\sigma} \tag{6}$$

c) Show that the elasticity of intertempral substitution

$$-\frac{\partial \log(c_1/c_2)}{\partial \log(1+r)} \tag{7}$$

is equal to $1/\sigma$. Interpret the intertemporal elasticity of substitution.

d) Now find the consumption function for c_1, relating the latter with y_1 and y_2

e) Show the effect on savings in period 1 of a change in the interest rate. Assume first that $y_2 = 0$, then show how the conclusions are influenced when y_2 is positive.

Exercise 4

Consider a consumer who lives for two periods, with incomes y_1 and y_2. The consumer can borrow at rate r_D, while she can save at the rate r_A, with $r_D > r_A$.

a) Show the budget constraint of the consumer in the space (c_1, c_2).

b) Determine the necessary and sufficient condition for the optimal consumption to be precisely (y_1, y_2). These conditions are in the form of inequalities of the derivatives of $U(c_1, c_2)$ evaluated at (y_1, y_2).

c) Suppose that the inequalities in b) are satisfied. Show graphically that if y_1 increases by a small amount Δy_1, then $\Delta c_1/\Delta y_1 = 1$ and $\Delta c_2/\Delta y_1 = 0$, which is closer to the keynesian consumption function. Explain.

d) Now suppose that the consumer cannot borrow at all, i.e. that $r_D = \infty$. Explain with a graph.

Exercise 5

Consider an individual who lives three periods ("young", "middle-aged" and "old"). Her incomes in the three periods are $Y_1 = Y$, $Y_2 = (1+\gamma)Y$, and $Y_3 = 0$. The individual wants to do perfect consumption smoothing, i.e. $C_1 = C_2 = C_3$. The interest rate is 0.

a) Compute savings in each periods.

b) Suppose that there is no population gowth, i.e. at each point in time one third of the population alive belongs to the three ages, i.e. one third earns Y, one third $Y(1+\gamma)$, and one thrid 0. What is aggregate savings in the economy?

c) Suppose that a pension system is introduced whereby each young and middle is forced to save A, and gets $2A$ when old. What happens to the savings of the individuals?

d) Now suppose that population grows at rate n each.period. Compute aggregate savings of the economy. Show how aggregate savings change when γ increases. Explain, and compare with your answer to b).

e) What is the rate of growth of aggregate income in this economy?

f) Does your answer support the often-heard statement that "in order to grow more a country has to save more"?