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## Problem set 2

These problems are taken from: José De Gregorio: *Macroeconomía*, Cap. 5, Pearson Education, Mexico City, 2007.

## Exercise 1

Consider an economy inhabited by an individual who lives two periods and has a utility function given by

$$U = \ln c_1 + \beta \ln c_2$$

where  $c_i$  is consumption in period i = 1, 2 and  $\beta = \frac{1}{1+\rho}$  is the intertemporal descount factor.

The individual earns  $Y_1$  and  $Y_2$  in period 1 and 2 respectively, and uses his income to consume, save and pay taxes. The real interest rate is r and the individual and government can lend or borrow at that rate.

Suppose the government spends G in period 1 and finances it entirely with a tax  $T_1$ .

(a) Express consumption and savings in each period as functions of income  $Y_1$  and  $Y_2$  and G.

(b) Suppose that the government wants to expand consumption in the first period and announces that it will not levy taxes in period 1 but will maintain spending at G, so that it will incur in a debt B. In period 2 it will impose taxes equal to  $T_2$ , consistent with its budget constraint. How does this affect consumption and savings in both periods? Is the government able to raise consumption in period 1? Compare with (a).

(c) Now suppose that fiscal policy is the same as in (a) and that the individual has liquidity constraints. In particular, assume that he can't borrow in period 1. Moreover, assume that:

$$Y_1\beta - G\beta < \frac{Y_2}{1+r}$$

Why is this constraint important? Compute consumption and savings in each period.

## Exercise 2

Suppose the utility function is  $u_t = \ln c_t + b(1 + L_t)^{1-\gamma}/(1-\gamma), b > 0, \gamma > 0$ and the period-t wage is  $w_t$ .

Consider the case where the individual lives only for one period and has no initial wealth. Solve the individual utility maximization problem. How does labor supply depend on the wage?

## Exercise 3

Suppose a government has public debt equal to 60% of GDP and is unable to repay its debt. Its creditors require that the ratio of debt to GDP be stable. The interest rate on debt is 10%. To meet the requirement, the government anticipates that with the above interest rate, a primary surplus equal to 4% of GDP, and a growth rate of 2%, the ratio of debt to GDP will remain stable in the following years, which will allow the interest on debt to fall.

(a) If the interest on debt falls, what happends to the surplus necessary to mantain a stable 60% debt to GDP ratio?

(b) Check that the calculations done by the government to meet stability are correct. (hint: given a growth rate  $\gamma$ , approximate  $1 + \gamma = 1$ ).

(c) What happens to the ratio of debt to GDP in the next 3 years if GDP starts to grow permanently at 4%?