Roberto Perotti

Problem set 2

These problems are taken from: José De Gregorio: *Macroeconomía*, Cap. 5, Pearson Education, Mexico City, 2007.

Exercise 1

Consider an economy inhabited by an individual who lives two periods and has a utility function given by

$$U = \ln c_1 + \beta \ln c_2$$

where c_i is consumption in period $i = 1, 2$ and $\beta = \frac{1}{1+\rho}$ is the intertemporal descount factor.

The individual earns Y_1 and Y_2 in period 1 and 2 respectively, and uses his income to consume, save and pay taxes. The real interest rate is r and the individual and government can lend or borrow at that rate.

Suppose the government spends G in period 1 and finances it entirely with a tax T_1.

(a) Express consumption and savings in each period as functions of income Y_1 and Y_2 and G.

(b) Suppose that the government wants to expand consumption in the first period and announces that it will not levy taxes in period 1 but will maintain spending at G, so that it will incur in a debt B. In period 2 it will impose taxes equal to T_2, consistent with its budget constraint. How does this affect consumption and savings in both periods? Is the goverment able to raise consumption in period 1? Compare with (a).

(c) Now suppose that fiscal policy is the same as in (a) and that the individual has liquidity constraints. In particular, assume that he can't borrow in period 1. Moreover, assume that:

$$Y_1\beta - G\beta < \frac{Y_2}{1+r}$$

Why is this constraint important? Compute consumption and savings in each period.

Answer:

(a) We know that $U = \ln c_1 + \beta \ln c_2$. Also

$$\begin{aligned} Y_1 &= c_1 + s + T_1 \\ c_2 &= Y_2 + (1+r)\, s \end{aligned}$$

Then the intertemporal budget constraint for the individual is

$$c_1 + \frac{c_2}{1+r} = Y_1 + \frac{Y_2}{1+r} - T_1$$

or

$$c_1 + \frac{c_2}{1+r} = Y_1 + \frac{Y_2}{1+r} - G$$

The FOC is

$$u'(c_1) = \beta (1+r)\, u'(c_2)$$

or

$$c_2 = \beta (1+r)\, c_1 \tag{1}$$

Substituting into the budget constraint we get

$$\begin{aligned} c_1 &= \frac{Y_1 - G}{1+\beta} + \frac{Y_2}{(1+r)(1+\beta)} \\ c_2 &= \frac{(1+r)\beta}{1+\beta}[Y_1 - G] + \frac{\beta Y_2}{1+\beta} \end{aligned}$$

since $s = Y_1 - c_1 - T_1 = Y_1 - c_1 - G$,

$$s = \frac{\beta [Y_1 - G]}{1+\beta} - \frac{Y_2}{(1+r)(1+\beta)}$$

(b) Now there is no taxation in period 1 but public speding G is financed by a tax T_2 in period 2. So:

$$\begin{aligned} Y_1 &= c_1 + s \\ c_2 &= Y_2 + (1+r)\, s - T_2 \end{aligned}$$

so the budget constraint of the individual becomes:

$$c_1 + \frac{c_2}{1+r} = Y_1 + \frac{Y_2 - G}{1+r}$$

The government must raise debt to finance speng in period 1, so clearly:

$$\begin{aligned} B &= G \\ T_2 &= (1+r)\, G \end{aligned}$$

The FOC for the individual's problem are the same as in (a), so substituting (1) into the new intertemporal budget constraint yields:

$$\begin{aligned} c_1 &= \frac{Y_1 - G}{1+\beta} + \frac{Y_2}{(1+r)(1+\beta)} \\ c_2 &= \frac{(1+r)\beta}{1+\beta}[Y_1 - G] + \frac{\beta Y_2}{1+\beta} \end{aligned}$$

We can see that the Ricardian equivalence holds, since consumption does not change in response to this change in policy (so neither does saving).

(c) The individual still maximizes

$$U = \ln c_1 + \beta \ln c_2$$

subject to the constraint

$$c_1 + \frac{c_2}{1+r} = Y_1 + \frac{Y_2}{1+r} - G$$

but also

$$c_1 \leq Y_1 - G$$

We can set up the Lagrangian

$$\mathcal{L} = \ln c_1 + \beta \ln c_2 + \lambda \left[Y_1 + \frac{Y_2}{1+r} - G - c_1 - \frac{c_2}{1+r} \right] + \mu [Y_1 - G - c_1]$$

The FOCs are:

$$\begin{aligned} \frac{d\mathcal{L}}{dc_1} &= \frac{1}{c_1} - \lambda - \mu = 0 \\ \frac{d\mathcal{L}}{dc_2} &= \frac{\beta}{c_2} - \frac{\lambda}{1+r} = 0 \\ \frac{d\mathcal{L}}{d\lambda} &= Y_1 + \frac{Y_2}{1+r} - G - c_1 - \frac{c_2}{1+r} = 0 \\ \frac{d\mathcal{L}}{d\mu} &= Y_1 - G - c_1 = 0 \end{aligned}$$

Now, if $\mu > 0$ it must be the case that $Y_1 - G = c_1$ (the borrowing co9nstraint is binding) so we can substitute into the third condition (the budget constraint) to find.

$$Y_2 = c_2$$

If $\mu = 0$ then $Y_1 - G - c_1 > 0$: the borrowing constraint is not binding, i.e. the individual cannot borrow but he would not want to borrow anyway. From conditions 1 and 2 we know

$$\begin{aligned} c_1 &= \frac{1}{\lambda} \\ c_2 &= \frac{1}{\lambda}\beta\left(1+r\right) \end{aligned}$$

so

$$c_2 = c_1\beta\left(1+r\right)$$

Substituting into the budget constraint yields:

$$\frac{Y_2}{1+r} + Y_1 - G = c_1\left(1+\beta\right) \tag{2}$$

but if the condition

$$\frac{Y_2}{1+r} > (Y_1 - G)\beta$$

holds, then by replacing in (2) we have

$$c_1 > Y_1 - G$$

which contradicts the condition $c_1 < Y_1 - G$ that must hold if the constraint is not binding. Hence, if $\frac{Y_2}{1+r} > (Y_1 - G)\beta$ the borrowing constraint must be binding. Intuitively, if the disposable income in period 2, Y_2, is large relative to the disposable income in period 1, $Y_1 - G$, then the individual would like to borrow to smooth consumption, and the constraint must be binding. Therefore:

$$\begin{aligned} c_1 &= Y_1 - G \\ c_2 &= Y_2 \end{aligned}$$

and obviously savings is 0.

Exercise 2

Suppose the period-t utility function, u_t, is $u_t = \ln c_t + b(1-L_t)^{1-\gamma}/(1-\gamma)$, $b > 0$, $\gamma > 0$ and the period-t wage is w_t. Consider the case where the individual lives only for one period and has no initial wealth. Solve the individual utility maximization problem. How does labor supply depend on the wage?

Answer:

We need to solve the household's one-period problem assuming no initial wealth and normalizing the size of the household to one. Thus the problem is given by:

$$\max_{c,L} \ln c + \frac{b(1-L)^{(1-\gamma)}}{1-\gamma}$$

subject to the budget constraint $c = wL$.
Set up the Lagrangian:

$$\mathcal{L} = \ln c + \frac{b(1-L)^{(1-\gamma)}}{1-\gamma} + \lambda(wL - c)$$

The first order conditions are:

$$\frac{d\mathcal{L}}{dc} = \frac{1}{c} - \lambda = 0 \tag{3}$$

and

$$\frac{d\mathcal{L}}{dL} = -b(1-L)^{-\gamma} + \lambda w = 0 \tag{4}$$

Substituting the budget constraint into equation (17) yields:

$$\lambda = \frac{1}{c} = \frac{1}{wL} \tag{5}$$

Substituting (19) into equation (18) yields:

$$-b(1-L)^{-\gamma} + \frac{w}{wL} = 0$$

and simplifying slightly:

$$\frac{1}{L} = \frac{b}{(1-L)^{\gamma}} \tag{6}$$

Although equation (20) only implicitely defines labor supply, we can see that it will not depend upon the real wage.

Exercise 3

Suppose a government has public debt equal to 60% of GDP and is unable to repay its debt. Its creditors require that the ratio of debt to GDP be stable. The interest rate on debt is 10%. To meet the requirement, the government anticipates that with the above interest rate, a primary surplus equal to 4% of GDP, and a growth rate of 2%, the ratio of debt to GDP will remain stable in the following years, which will allow the interest on debt to fall.

(a) If the interest on debt falls, what happends to the surplus necessary to mantain a stable 60% debt to GDP ratio?

(b) Check that the calculations done by the goverment to meet stability are correct. (hint: given a growth rate γ, approximate $1 + \gamma = 1$).

(c) What happens to the ratio of debt to GDP in the next 3 years if GDP starts to grow permanently at 4%?

Answer:

(a) If the interest rate falls, then ceteris paribus a smaller surplus is necessary to keep the ratio debt to GDP stable. In fact from

$$b_{t+1} - b_t = -\frac{s_t}{1+\gamma} + \frac{r-\gamma}{1+\gamma} b_t \tag{7}$$

in steady state

$$b_{ss}(r - \gamma) = s_{ss}$$

(b) From (7) substituting the figures given and approximating:

$$b_{t+1} - b_t \simeq -4\% + 8\% \, (60\%) \simeq 0.8\% > 0$$

Therefore with a surplus of 4% and the given interest and growth rate the debt to GDP ratio is going to increase and NOT BE STABLE!

(c)

$$b_1 - b_0 = -4\% + 3.6\% = -0.4\%$$

with these new conditions, the debt to GDP ratio starts to decrease and it will do so in the following years as well.